A New Social Network Model of Online Forums

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***Abstract*—Modeling online opinion dynamics plays an important role toward in-depth comprehension of collective interactive behavior in modern human society and future digital society. Some statistical models based on network science or social networks have been known for years. In this paper, we propose a novel network model of online forums. Different from the existing models in which model selection is obscure, we accommodate reasoning in both mathematical and psychological contexts such that the basic parameters of the network model can be identified in an intuitive way. The proposed first-order-aging-with-fitness (FOAF) model extends from the well-known BA model, by adjusting weights for younger nodes, to better reflect the truth of Internet forums. We successfully verify the FOAF model and consequent entire social network model of wider applicability and better alignment with real online opinion data.**

I. Introduction

Online social networks and interactive Internet forums have initiated a revolution in social awareness. Public can easily access any news and event, share their own opinions, and even interact with authors immediately. This fact changes the landscape of social movements, like sharing ideas of a movement, gathering people to a protest and raising funds for any campaign. All activities, which used to be launched through face-to-face contacts, can now be held in the cyberspace without the limit of time and locations [1], which creates significant interests in understanding collective dynamic behavior over Internet, known as *information dynamics* in [2].

Although *information dynamics*, which comprise the information dissemination and impacts on public reaction, is believed to expound the success of a social movement [3], it remains open to understand the mechanism of information dynamics and their relationships with social movements. Unlike conventional media, nowadays information is no longer furnished in one direction but disseminates bidirectionally between Internet users, evolving by lots of clicks and typing. Moreover, after information dynamics were identified to lead to several collective actions in many social movements [4], [5], it is likely that public reaction to the information primarily leads to the consequence of a movement. A movement can lodge an on-street protest or end without further attention after most users reaching a consensus or their opinions distinct from one another, respectively. Furthermore, the mechanism of information dissemination significantly contributes to the success of a social movement. Although study proceeds such as Arab Spring [6], the complete analytical comprehension of online social behavior remains one of the most wanted intellectual challenges. Broader impacts on e-commerce and marketing are expected from such knowledge.

Observe that most online opinion dynamics share some common characteristics: the opinion of a person is affected by his neighbors and opinions may reach a consensus. Many Markov-based models have been proposed. Early works among them are Degroot model [7] and Voter model [8]. It has been found that stubbornness [9] can change the outcome of opinion dynamics. More interestingly, normal agents may become stubborn during opinion exchanges [10]. In a similar manner, latent states [11], [12] can change the outcome of opinion formation. Still other studies incorporate repulsion through negative ties, forming a model with no global consensus but group polarization [13]. Moreover, it has been shown that network structures strongly affect the outcome in a trust-distrust relationship [14]. Further efforts such as opinion dynamics [15], learning mechanism [16], multi-dimensional modeling [17] and consensus under multiple opinions leaders [18] have been published very recently. However, beyond data fitting, the effective network model to delineate the collective behavior of massive interactive agents remains an open knowledge.

Consequently, on top of the past remarkable results on network science [19], further network analysis [20], [21] and social learning [22], we re-visit the network model of online forums in this paper, such that more effective inference of such complex dynamic networked system could be more possible as existing network models pretty much describe the statistics of data only. First of all, we note that the well known preferential attachment mechanism in scale-free networks [23] are based on statistics, but further dynamics insights and psychological factors are not well taken into consideration. We therefore design a novel mechanism that is still mathematically consistent with results from scale-free phenomenon. A more convincible network model is delightfully established. We further verify the model with appropriate datasets suggesting quasi-closed social networks to avoid bias from partial social networks. We compare two different online forums: data collected from PTT about a famous large social movements took place in 2014 in Taiwan, the protest against the Cross-strait Service Trade Agreement (CSA), and the data collected from InsideHoops about basketball games. These meaningful data set provide temporal and dynamic network structures bearing the keys to fundamental modeling. The experimental results pretty much agree on our proposed model that fits better than the well known BA model in degree distributions and other statistics.

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II. Scale-Free Network Models

The scale-free phenomenon on networking structure and distributions have been widely known since the birth of network science [19], and can be explained by the preferential attachment. However, the universality of scale-free topology does not necessarily imply the growth of networks follow the same process. Conversely, studies have shown that various elementary processes can be introduced to network models [23], and all of these variations still lead to scale-free networking structure with different scaling exponents.

The BA (Barabási Albert) model has been widely used for nearly 20 years [19]. It takes each edge's attachment probability to node i as $\Pi_i = k_i / \sum_j k_j$, where k_i denotes i's degree, so as to generate a scale-free network with scaling exponent -3. Furthermore, the Bianconi-Barabási model was suggested to considering the fitness of node into modeling so that individual differences is considered, which is also known as fitness model [24]. Inspecting realistic data such as Section V, these models clearly have room to improve.

Other elementary mechanisms such as aging [25] [26], node deletion [27], accelerated growth [28], internal links and initial attractiveness [29] are introduced to enrich the diversity and universal applicability of BA model. However, when online opinion dynamics plays an important role in e-commerce and even politics, in this paper, we intend to take psychological factor into consideration. We note that online readers tend to read newer articles and comments, so aging is obviously a critical issue to model opinion dynamics. Taking aging into modeling, the attachment probability now turns into

$$\Pi_i = \frac{k_i (n - n_i)^{-\alpha}}{\sum\limits_j k_j (n - n_j)^{-\alpha}}, \tag{1}$$

where $\alpha > 0$ is the parameter for aging, and $n - n_i$ is the age of node i.

Nevertheless, appropriate identification of elementary mechanism(s) is not easy due to unknown mixture of them. It is even more complex by fitting realistic data. For example, in the aging model, the magnitude of aging parameter is unclear, and its growth still lacks of proper interpretations and subsequent polishing in general applications, particularly to model human collective behaviors.

III. Main Results

Although BA model succeeds in explaining certain statistics on data, it serves as the basic model and is difficult to infer or even to predict along the time evolving process. To more effectively model online opinion dynamics (authoring many articles and even more comments), such that inference or control of such complicated dynamic networked systems is possible, an appropriate model is very much wanted.

A. Problem Formulation

The network structure on online forums can be viewed as an randomly generated bipartite network composed of authors and commenters. More preciously, under the discrete time fashion, supppose the time step n corresponds to the nth arrived comment. At each time step, we need to know the creation of new author/commenter and the pair of author and commenter that the new comment connects to.

In this paper, the creation of a new author/commenter is taken as a Bernoulli random process. Our main focus is to model the connection between authors and commenters, which is equivalent to find the probability that the nth arrived comment connects to a pair of author and commenter. However, directly identifying a joint distribution over authors and commenters is too hard to be generally applicable, so we approximate it as the product of marginal distributions. That is, the problem boils down to properly identify the marginal distributions of authors and commenters being connected to the nth arrived comment.

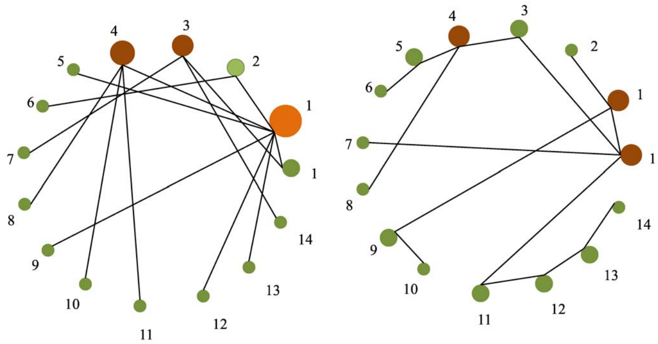
B. First-Order-Aging-with-Fitness Model

By considering psychological aspects of Internet user behaviors, we propose a novel model called First-Order-Aging-with-Fitness (FOAF) model, whose intuitive comparison with BA model is depicted in Fig. 1. Instead of disregarding ages and just considering statistics of node degrees as BA model, FOAF model tends to favor young nodes with higher weights. Once a new attraction (say, a super star in a social network) emerges, the popularity of the old nodes fade. Therefore, though both models produces power law degree distribution, FOAF model has a lower magnitude of scaling exponent, making the degree distribution more uniform. Moreover, the advantage of FOAF model over aging model is that it is more intuitive with respect to the growth of nodes. [25] showed that the the growth of nodes in aging model is described by some mixture of exponential, digamma and hypergeometric functions, so the engineering meaning is vague and hard to directly generalize to meet our purpose. As a contrast, in section III.C, we will show that in FOAF model, the growth of nodes follows a first order differential equation, and the solution is a age-driven and fitness-dependent growth with a decay constant regarding to the competition among nodes.

More precise in the grand picture of modeling online opinion dynamics, the FOAF model is accounted for authors of the articles due to competition and individuality among them and the Bianconi-Barabási model is employed to stand for their autonomy and independence. Then, we propose a transformation from discrete time to continuous time, describing the growth of nodes on the continuous time line.

C. Generative Models

In an online forum, there are authors and commenters. An author writes articles to attract comments from commenters. And a commenter comments on articles according to their own will. Thereby, the models for authors and commenters are fundamentally different. To account for such difference, we propose the following models on discrete time n, where each time step n correspends to the nth arrived comments.



(a) BA model, $\Pi_i \propto k_i$

(b) FOAF model (same fitness), $\Pi_i \propto k_i \frac{n}{n-n_i}$

Figure 1: **Comparison of formation between (a) BA model and (b) FOAF model under the same fitness in undirected random networks.** The numbers denote the creation times. The area of a node is proportional to its degree. Because in real social networks, the popularity of old nodes fade once a new super star emerges, the major advantage of FOAF model over BA model is that it tends to give young nodes higher weights. Hence the edge tends to exist between two nodes with similar creation times, and the degree distribution is more uniform than BA model; i.e., FOAF model has a lower scaling exponent. Note that in this picture we assume all nodes' fitness are equal. Even with different fitness, FOAF model still tends to favor young nodes with higher weights.

Authors should experience decays of popularity because they always compete with each other, implying an aging process. Accordingly, we define the order of aging as:

Definition 1: An aging process is $p-$order if $\alpha = 1$ in eq. (1).

Since attachment probability is $\Pi_i = \frac{\partial k_i}{\partial n}$, it would be more suitable in unit if we take $\Pi_i \propto k_i/(n-n_i)$. Thereby we take the first-order aging ($\alpha = 1$ in eq. (1)) to model the decay. Besides, each author has his/her distinct novelty, so we adopt fitness for their individual abilities to attract comments. More details will be elaborated in the following paragraphs. These two rationals give First-Order-Aging-with-Fitness (FOAF) Model.

Proposition 1 (FOAF Model For Authors): The attachment probability of author i follows

$$\Pi_i = \frac{\xi_i k_i \frac{n}{n-n_i}}{\sum_j \xi_j k_j \frac{n}{n-n_j}} \qquad (2)$$

$$\xi \sim \mathsf{Beta}(a, b),$$

where k_i, ξ_i and n_i are the degree, fitness and creation time of author i. The factor $\frac{n}{n-n_i}$ is for first-order aging. Note that the BA model does not possess an explicit time-dependent factor such as the first-order aging.

On the other hand, commenters can leave comments as many as they like. They are less subject to competition, so we do not adopt aging for commenters. Still, different commenters should have different initiatives to make a comment. Thereby, due to autonomy and individuality, Bianconi-Barabási model, or the fitness model, appears suitable for commenters.

Proposition 2 (Bianconi-Barabási Model For Commenters): The attachment probability of commenter j follows

$$\Pi_j = \frac{\beta_j k_j}{\sum_m \beta_m k_m} \qquad (3)$$

$$\beta \sim \mathsf{Beta}(c, d),$$

where k_j and β_j are degree and fitness of commenter i.

The origins of attachment probability for authors and commenters are different. For author i, Π_i is the probability that i attracts a comment, while for commenter j, Π_j is the probability that j yields a comment. With a more careful inspection, for author i, $\Pi_i \propto k_i$ means that his ability to attract comments is proportional to his cumulative popularity, which is the principle of the BA model. In contrast, for commenter j, $\Pi_j \propto k_j$ means that his ability to yield comments is proportional to his cumulative productivity, following Yule's process for biotic organisms [30].

The intuitive interpretations of the fitness for author ξ and the fitness for commenter β can be summarized as follows. Generally speaking, the fitness of a network node can be viewed as its intrinsic tendency to attract link connections, according to various situations. ξ_i is regarded as the probability that a commenter leaves a comment after reading the author i's articles. In a different manner, β_j is taken as the relative amount of time (from 0 to 1) that commenter j spends on the forum, because the more time one spends on the forum, the more likely that one yields a comment. Under this proposition, suppose an author has fitness ξ and retains m distinct commenters' visits. Then the number of comments the author got follows $\mathsf{Binomial}(m, \xi)$. Conversely, suppose ξ follows a prior distribution $p(\xi)$. The posterior distribution given the author got k comments in m visits, $p(\xi|k, m)$, may follow the same familiy as $p(\xi)$. Hence, ξ is chosen as the *conjugate prior* [31] of Binomial distribution, and the answer is the Beta distribution. The same principle applies for β. As a result, we take $\xi \sim \mathsf{Beta}(a, b)$ and $\beta \sim \mathsf{Beta}(c, d)$.

With the generative models for authors and commenters, a network model of the online forum is described as follows. At each discrete time n, a new comment arrives, a new author is created with probability p and a new commenter is created with probability q. This arriving comment connects to an author following the FOAF model and to a commenter following the Bianconi-Barabási model.

D. Growth Models

Here we analyze the growths of authors' and commenters' degrees in the network model. In fact, the evolution of commenters' degrees has already been derived as the following.

Fact 1 (Biaconi-Barabási): The growth of commenter j's degree k_j is

$$k_j(n; n_j, \beta_j) = (n/n_j)^{\beta_j} \qquad (4)$$

From eq. (4), a node grows according to its fitness. And node j's penalty of late creation is $n_j^{-\beta_j}$. Since it does not depend on other nodes, node j is autonomous.

The next step is to specify the growth of authors' degrees. Since FOAF model includes fitness, the growth of degree in FOAF model may be similar to that of Bianconi-Barabási model like $k_j = (n/n_j)^{\beta_j}$. However, FOAF model further considers aging, and thus $n - n_i$, the age of node i, could be more important than n, the age of the network. Plus, the FOAF model considers competition among nodes (resulting in aging), so the penalty of late creation should depend on others. Therefore, in FOAF model, $k_i \propto n_i^{-\eta}(n-n_i)^{\xi_i}$, where η is a jointly determined parameter for late creation penalty. The next proposition describes this consideration.

Proposition 3: The growth of author i's degree k_i follows

$$k_i(n; n_i, \xi_i) \propto n_i^{-\eta}(n-n_i)^{\xi_i}, \text{ for some } \eta \in (0,1) \tag{5}$$

To show that eq. (5) is a satisfiable solution to author's generative model eq. (2). It suffices to show that the summation of degrees equals to n and that eq. (5) is a solution to eq. (2).

Since the total degree should always be n, we shall find a condition of stabilizing the total degree. Suppose at each discrete time step, a new author is created with probability p. Let $f(\xi)$ be the probability density function of ξ. Then, the summation of degrees becomes

$$\begin{aligned}
\mathbb{E}\Big[\sum_i k_i\Big] &\propto \int_0^1 f(\xi)d\xi \int_1^n n_i^{-\eta}(n-n_i)^{\xi} p dn_i \\
&\overset{n\to\infty}{=} \int_0^1 pn^{\xi-\eta+1} f(\xi)d\xi \int_0^1 \Big(\frac{n_i}{n}\Big)^{-\eta}\Big(1-\frac{n_i}{n}\Big)^{\xi} d\Big(\frac{n_i}{n}\Big) \\
&= \int_0^1 pn^{\xi-\eta+1} \frac{\Gamma(1-\eta)\Gamma(\xi+1)}{\Gamma(-\eta+\xi+2)} f(\xi)d\xi \\
&= \Theta(n), \quad \text{for some } \eta \in (0,1)
\end{aligned} \tag{6}$$

As a result, there exists an η such that the summation of degrees scale with n. In addition, since in eq. (5) we only assume $k_i(n; n_i, \xi_i)$ to be proportional to $n_i^{-\eta}(n-n_i)^{\xi_i}$, the model could surely be normalized such that the summation of degrees equals to n.

After stabilizing the total degree, we shall check whether the proposed model is a solution. First, simplify eq. (2) into

$$\frac{\frac{\xi_i}{n-n_i}k_i}{\sum\limits_j \frac{\xi_j}{n-n_j}k_j} \tag{7}$$

Note that in the discrete time

$$\Pi_i = \frac{\partial k_i}{\partial n} \overset{eq.\ (5)}{=} \frac{\xi_i}{n-n_i}k_i \tag{8}$$

As long as the denominator of eq. (7) is a constant, we are done. Thereby, consider the expectation of the denominator, the proof is completed by

$$\mathbb{E}\Big[\sum_j \frac{\xi_j}{n-n_j}k_j\Big] \overset{eq.\ (8)}{=} \mathbb{E}\Big[\sum_j \frac{\partial \xi_j}{\partial n}\Big] \overset{eq.\ (6)}{=} \Theta(1) \tag{9}$$

Remark 1: η, representing the extent of competition among authors, is implicitly determined in eq. (2).

Because in the generative model, the summation of degrees always equals to n, the network is automatically stabilized, and the nodes jointly determine η. In addition, η is the extent of competition among authors because $f(\xi)$ represents the distribution of author's fitness. Intuitively, the competition among authors becomes more intense if $f(\xi)$ favors large ξ. Mathematically, η becomes higher if $f(\xi)$ favors large ξ. Hence, η can represent the extent of competition.

E. Continuous Time Generative Model

Since realistic social/opinion networks evolve on the continuous time line, we should transform our discrete time model to a continuous time version. Accordingly, we extend the domain of n and k_i from $\mathbb{N}$ to $\mathbb{R}_+$. Let $\tilde{n}$ and $\tilde{k}_i$ be the extensions of discrete time and node i's degree. In either Bianconi-Barabási model or FOAF model, the only time-dependent parameter is $\tilde{n}$, so the growth rate of node i at time t is

$$\frac{\partial \tilde{k}_i}{\partial t} = \frac{\partial \tilde{k}_i}{\partial \tilde{n}}\frac{\partial \tilde{n}}{\partial t} \tag{10}$$

Due to eq. (6), $\sum\limits_j \frac{\partial \tilde{k}_j}{\partial \tilde{n}}$ can be normalized to 1. Thus,

$$\frac{\partial \tilde{k}_i}{\partial t} = \frac{\frac{\partial \tilde{k}_i}{\partial \tilde{n}}\frac{\partial \tilde{n}}{\partial t}}{\sum\limits_j \frac{\partial \tilde{k}_j}{\partial \tilde{n}}\frac{\partial \tilde{n}}{\partial t}}\frac{\partial \tilde{n}}{\partial t} = \frac{\frac{\partial \tilde{k}_i}{\partial t}}{\sum\limits_j \frac{\partial \tilde{k}_j}{\partial t}}\frac{\partial \tilde{n}}{\partial t} \tag{11}$$

Equation (11) means that a continuous time generative model origins from a discrete time generative model embedded on a mapping of $\tilde{n}$ and t.

To elaborate further, suppose there is an edge appears right after time t, or at t^+. Then the probability that i is selected is proportional to $\frac{\partial \tilde{k}_i}{\partial t}$. Hence, by normalization, $\frac{\frac{\partial \tilde{k}_i}{\partial t}}{\sum_j \frac{\partial \tilde{k}_j}{\partial t}}$ is the attachment probability at time t^+. With this fact, we find a distinction of continuous time and discrete time. The attachment probability $\frac{\frac{\partial \tilde{k}_i}{\partial t}}{\sum_j \frac{\partial \tilde{k}_j}{\partial t}}$ can be specified by the discrete time model. And the arrival rate $\frac{\partial \tilde{n}}{\partial t}$ is the only new thing that is introduced in the continuous time model, so upon specifying it, the model is successfully transformed from discrete time to continuous time.

IV. Data Sets

Identifying appropriate datasets to verify network modeling is never easy. Many partial data from large-scale online social networks might suffer bias from sampling. To completely characterize the dynamics and thus, to specify the model, datasets from quasi-closed social networks are preferred, and better with severe or special collective behaviors to verify. Luckily, we obtain two quite different data sets across cultures for our purpose.

A. Social Information Platform: PTT

PTT is a terminal-based bulletin board system (BBS). It is not only the largest online message board in Taiwan but also the dominating online platform of opinions. Therefore, most public affairs can rise up lots of discussions on PTT such that anyone can observe any movement based on the data.

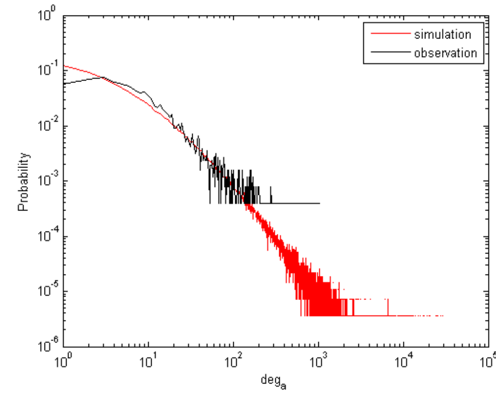
More importantly, the regulations of posting and commenting articles on PTT are very clear to avoid random behaviors. New users cannot post articles unless they satisfy some requirements, like going online for at least 30 days after registration. Contents of articles and comments are examined by the manager of each thread to filter meaningless ones. With its rigorous regulations, the number of users acting on PTT is almost fixed during a period of time. Hence, we can analyze information dissemination conveniently by taking PTT as a quasi-closed network.

The incident we study on PTT is a social movement shocked the Taiwan public in 2014: the protest against Cross-strait Service Trade Agreement (CSA), inducing large numbers of people, from students to retired seniors, to rethink public affairs, pushing Taiwan to move on the road of democracy, not oligarchy. The CSA incident provides us great data to uncover the pattern characterizing online information dynamics. We collect all articles related to CSA, including the comments of partially anonymous users identified by user ID only.

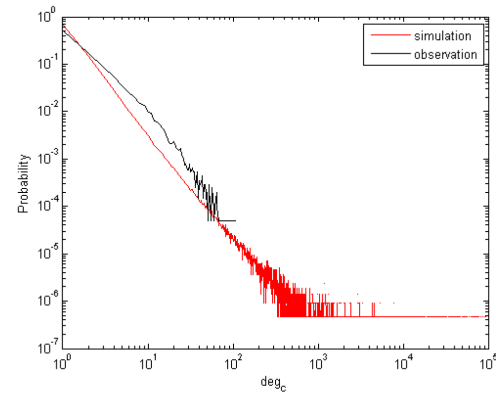
B. Basketball Information Platform: InsideHoops

In contrast to PTT, InsideHoops is the web's leading NBA basketball site. The regulations of posting and commenting articles are softer than that of PTT. In fact, it only takes a few minutes for newcomers to register, so InsideHoops encourages all basketball fans to join, share and have fun.

Just because all users are basketball fans, compared with PTT, the culture in InsideHoops is more homogeneous, the distances among users are shorter and users are earger to leave comments repeatedly. Clearly it is a quasi-closed network. These features make InsideHoops a good website to study the long term statistics in a quasi-closed system. Therefore, inspite of observing a specific event in a short period of time as we do on PTT, we crawled all articles from 2013 to 2017 to get the knowledge of information dynamics on InsideHoops.



(a) Authors on PTT.



(b) Commenters on PTT.

Figure 2: **The observed and simulated degree distributions of PTT.** (a) author; (b) commenter. The simulation almost matches the actual distribution and outperform existing models.

V. Experimental Verifications

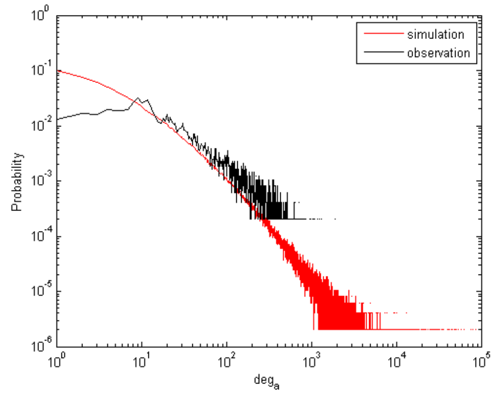
From the simulation shown in Fig. 2, our model captures the general shape of PTT's degree distribution. However, there is a critial problem: The degree of the highest degree node is too high. This fact implies a criterion for authors/commenters to stop receiving/yielding comments. For example, after posting articles, an author enters a "popular state" in which he constantly receives comments. Once the author meets certain stopping criterions, he may transit to an "outdated state" in which he gains virtually no comment. The same idea applies for commenters. Hence a stopping criterion is very wanted.

Comparing Fig. 2 and Fig. 3, we note that Bianconi-Barabási model can only capture the commenter's degree distribution on PTT, not on InsideHoops. Recall that the users of PTT are from the general public while that of InsideHoops are from basketball fans. If a commenter is sampled from the PTT, it is possible that this commenter just watches the news and not going to do commenting on the website, such indifference or apathy of the public has been recognized for a long time [32]. On the other hand, the enthusiasm of basketball fans can trigger commenting. Thus, the magnitude of commenter's scaling exponent on PTT is larger than that on InsideHoops because the general public is more indifferent than basketball fans. Plus, typical examples for Bianconi-Barabási are citation networks and web documents [23]; therefore, Bianconi-Barabási may be more suitable for the general public than for groups of fans.

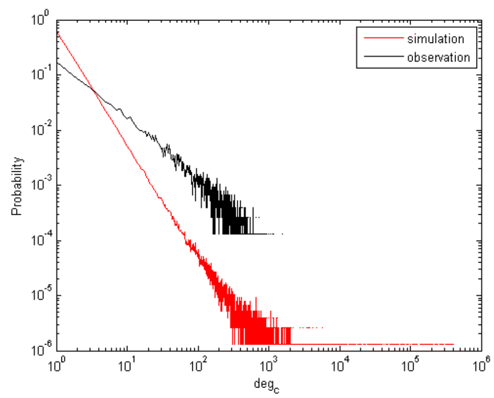
Finally, from Fig. 2 and Fig. 3, the FOAF model captures the tail distributions of authors on PTT and on InsideHoops. Note that the BA model cannot have this feature, because its scaling exponents is around 2 to 3, but PTT and InsideHoops's are around 1 to 2. The partial success of the FOAF model implies that the roles of authors are basically the same on both forums. Either among general public or basketball fans, an author needs to await comments and even compete with others. Hence we observe the accordance.

VI. Conclusion

This paper provides a novel network model for online forums. With psychological and mathematical insights, we propose the FAOF model for authors and the Bianconi-Barabási for commenters. Combining both models, a discrete time network model is established. Extending to continuous



(a) Authors on InsideHoops.



(b) Commenters on InsideHoops.

Figure 3: **The observed and simulated degree distributions of InsideHoops.** (a) author; (b) commenter. The figures show that the FOAF model can explain the distribution of authors, but Bianconi-Barabasi model cannot explain the distribution of commenters.

time models, we note that a continuous time model can be described by the attachment probability and the arrival rate. Finally, we find that the FAOF model is partially suitable for authors on PTT and InsideHoops, while the Bianconi-Barabási model is only applicable for commenters on PTT. Thus, the behaviors of authors are similar in both forums, while that of commenters are quite different.

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